WOODIN

Investigation of a Steel Highway Bridge

Civil Engineering

B S

1906

INVESTIGATION

OF A

STEEL HIGHWAY BRIDGE

BY

EARLE BELMONT WOODIN

THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

PRESENTED JUNE 1906

U N I V E R S I T Y O F I L L I N O I S

May 24, 1906

This is to certify that the thesis prepared under the immediate direction of Assistant Professor F. O. Dufour by

EARLE BELMONT WOODIN

entitled INVESTIGATION OF A HIGHWAY BRIDGE

is approved by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

Ira O. Baker.

Head of Department of Civil Engineering

Location

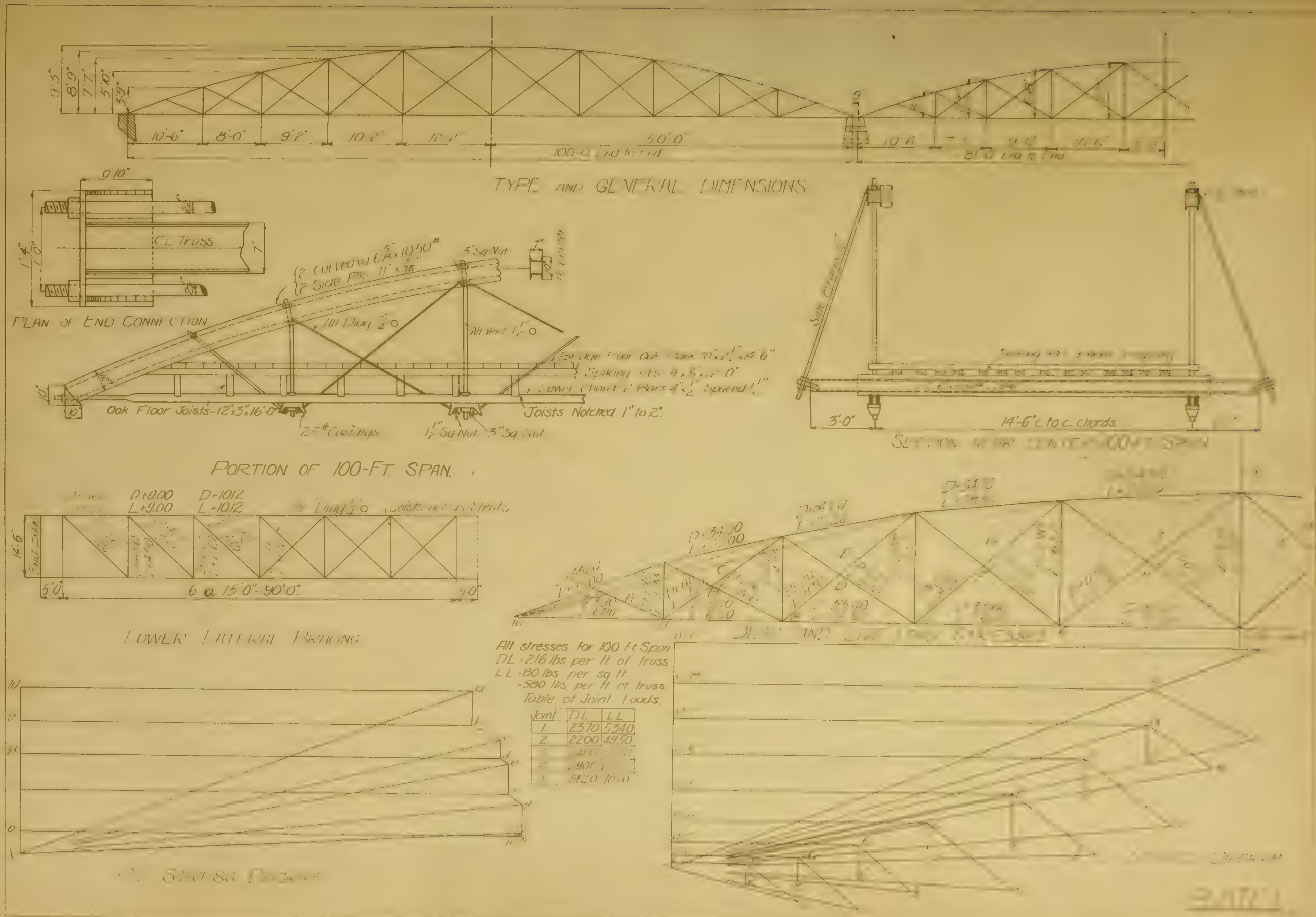
The highway bridge which is the subject of this investigation, is located on the State Road over Salt Fork Creek, $1\frac{1}{2}$ miles south of St. Joseph, Illinois.

Description.

The bridge is a wrought iron structure, built in 1867 by Z. King and Co. Cleveland, Ohio, and consists of one 100-ft. span and one 85-ft. span. The truss is of the bowstring type. The form and dimensions of the truss, together with the construction of the details are shown on Plate I. The foundation consists of two abutments and one pier, each constructed of soft sandstone masonry.

The Reasons for Investigation.

Upon examination of this bridge Professor Ira O. Baker condemned it. This



TYPE AND GENERAL DIMENSIONS
100-0 End to End
50'0"
10'-6"
8'-0"
9'2"
10'2"
9'5"
8'9"
7'7"
5'10"
3'9"
PLAN OF END CONNECTION
CL Truss
1'4"
1'0"
0'10"
2 Curved 6 3/8 x 10.50#
2 Side Pls. 11" x 7/16
5" Sq. Nut
All Diag 3/4 O
All Vert 1 1/2 O
Oak Floor Joists-12"x3"x16'-0"
Joists Notched 1" to 2"
Lower Chord 2 Bars 4" x 1/2" Spaced 1"
25# Castings
1 1/2" Sq. Nut
3" Sq. Nut
PORTION OF 100-FT. SPAN.
14'-6" c. to c. chords.
3'-0"
D+9.00
L+9.00
D+10.12
L+10.12
6 @ 15'0" = 90'0"
5'0"
14'6"
LOWER LATERAL BRACING
All stresses for 100 Ft. Span
DL = 216 lbs per ft. of truss
LL = 80 lbs per sq. ft.
= 580 lbs. per ft. of truss.
Table of Joint Loads.
Joint DL LL
1. 2370 5340
2. 2200 4950
D+34.00

investigation is made to determine the weakest members and their various efficiencies.

Extent of Investigation.

The investigation will be confined almost wholly to the 100-ft. span, since the size of corresponding members in the two spans are approximately equal and therefore the longer span is more unsafe than the shorter one.

Computation of Weight.

The weight of the metal was assumed to be 480 lb. per cubic foot, and the weight of the lumber was assumed to be $4\frac{1}{2}$ lb. per foot B.M. The computation follows in tabular form.

TABLE I.
WEIGHT OF METAL IN 100-FT. SPAN.

1	2	3	4	5	6	7	8	9.
Ref No.	Name of Piece.	No. of Pcs.	Dimen. of Cross Section.	Length in Ft. & In.	Wt. per Ft. lb.	Wt. of Main Mem. lb.	Wt. of Det. lb.	% Det. of M. Mem.
1.	Top Chord.							
	Channels	4	6" x 10.50#	102'-0"	10.50	4,284		
	Side Pls.	4	$\frac{5}{16}$" x 11"	102'-0"	11.69	4,768		
	Splice Pls.	26	$\frac{5}{16}$" x 9"	11"	9.56		214	
	" "	1	$\frac{5}{16}$" x 11"	3'-0"	11.68		35	
	" "	7400	$\frac{5}{8}$" ⊗	per 100'	11.1		820	
						9,052	1,069	11.8
2.	Lower Chord.							
	Bars.	4	4" x $\frac{1}{2}$"	100'-0"	6.80	2,720		
	Splice Pls.	20	4" x $\frac{1}{2}$"	0'-11"	6.80		156	
	Rivet Hds.	240	$\frac{3}{4}$" ⊗	per 100	16.1		39	
						2,720	195	7.2
3.	Verticals.							
	Rods.	4	$1\frac{1}{2}$" O.	5'-3"				
	"	4	$1\frac{1}{2}$" O.	7'-6"				
	"	4	$1\frac{1}{2}$" O.	9'-3"				
	"	4	$1\frac{1}{2}$" O.	10'-3"				
	"	4	$1\frac{1}{2}$" O	10'-9"				
	Totals Ford.	4		43'-0"		11,772	1,264	

Table I-Continued.

1	2	3	4	5	6	7	8	9.
	Forw'd.					11,772	1,264	
3	Verticals (cont.)	4	$1\frac{1}{2}$" O	43'-0"	6.01	1,050		
	Nuts	36	$2\frac{3}{4}$" □	per 100	243.9		88	
	Washers.	36		per 100	36		14	
						1,050	102	10.0
4	Diagonals.							
	Rods	4	$\frac{3}{4}$" O	6'-0"				
		4		9'-6"				
		4		10'-6"				
		4		11'-3"				
		4		12'-6"				
		4		13'-3"				
		4		14'-0"				
		4		15'-6"				
		4		16'-0"	1.50	711		
	Nuts.	36		per 100	41.00		15	
5	Sp. Castings.	18	2"x 3"	1'-3"	25.00		450	
						711	465	65.3
	Side Bracing for Top Chord.							
	Rods.	4	$1\frac{1}{2}$" O.	8'-0"	6.01	192		
	"	2	$1\frac{1}{2}$" O	9'-6"	6.01	114		
	Channels, Fl.	3	6"x 10.50	20'-0"	10.50	630		
	Forw'd.					13,533	1,831	

Table I-continued.

1	2	3	4	5	6	7	8	9
	Fo'rd.					13,533	1,831	
5	Side Bracing (Fo'rd.)					936		
	Rivet Hds.	72	$\frac{3}{4}$" o	per 100	16.1		12	
						936	12	1.3
6	Lower Lateral Bracing.							
	Rods.	12	$\frac{3}{4}$" o.	21'-0"	1.50	376		
	Bolts & Nuts	24	$\frac{1}{2}$" o	per 100	22.50		5	
	Hoop Circle	6	each		3.00		18	
						376	23	6.0
7	Spikes	1400	$\frac{1}{4}$"x5" wrt.	per 100	7.70		108	
8	End Shoes.	4	each		100		400	
	Totals					14,845	2,374	16.0

Total weight of main members = 14,845 lb.
Total weight of Details = 2,374 lb.
Total weight of Metal-100-Ft. Span = 17,219 lb.

TABLE II.

WEIGHT OF LUMBER IN 100-FT. SPAN.

Ref. No.	Name of Piece	No. of Pcs.	Dim. of Cross Section	Length in Ft.	No. of Ft. B.M.	Wt. per Ft. B.M. lb.	Wt. of Pcs. lb.	Total Wt. of Lumber lb.
1.	Joists.	51	12"x3"	16'-0"	2,450	4.50	11,000	
2.	Stringers.	15	3"x4"	100'-0"	1,500	4.50	6,750	
3.	Flooring.	100	12"x2½"	14'-6"	3,620	4.50	16,300	
								34,050

SUMMATION OF WEIGHTS IN 100-FT. SPAN.

Total weight of metal in span.-lb.	17,219
Total weight of lumber in span.-lb.	34,050.
Total weight of 100-Ft. Span.-lb.	51,269

$\frac{51,269}{100}$ = 513 lb. per lineal ft. of span.

$\frac{513}{2}$ = 256.5 lb. per lineal ft. of truss.

Determination of Stresses

The stresses were determined by both graphical and analytical methods. Two conditions were considered; 1) assuming that the top chord is hinged at the joints and 2), assuming that the top chord acts as an arch rib.

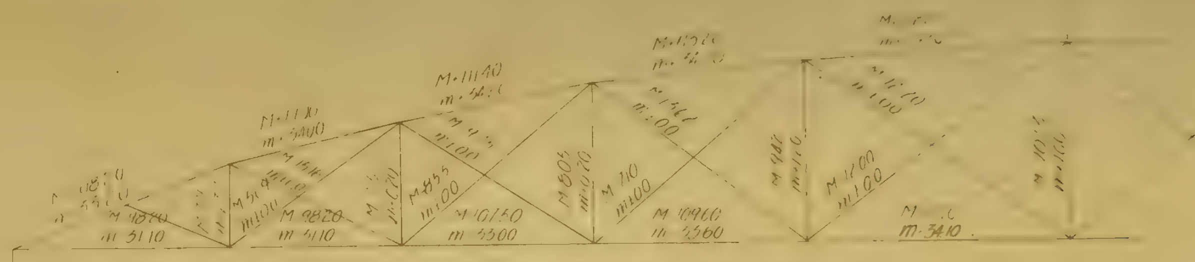
The Top Chord with Hinged Joints.

In this case the structure is assumed to act as a pair of simple trusses supporting a system of vertical loads. The top chord is considered as being straight between panel points. The dead load at the panel points was computed from the results obtained in the summation of weights, p. The original dead load stress diagram was drawn to a scale of 2,000 pounds per inch. The dead load stresses are placed on a diagram of the truss, Plate I. The live load at the panel points was computed for a uniform

load of 80 lb. per square foot of floor surface. This is the conventional live load for which modern highway-bridges of similar span are designed. The original live load stress diagram was drawn to a scale of 5,000 pounds per inch. The stresses in the verticals was assumed to be equal to the maximum joint load. The live load stresses are placed on a diagram of the truss, Plate I. The maximum and minimum stresses for this solution are placed on a diagram of the truss, Plate II.

The Top Chord Considered as an Arch Rib.

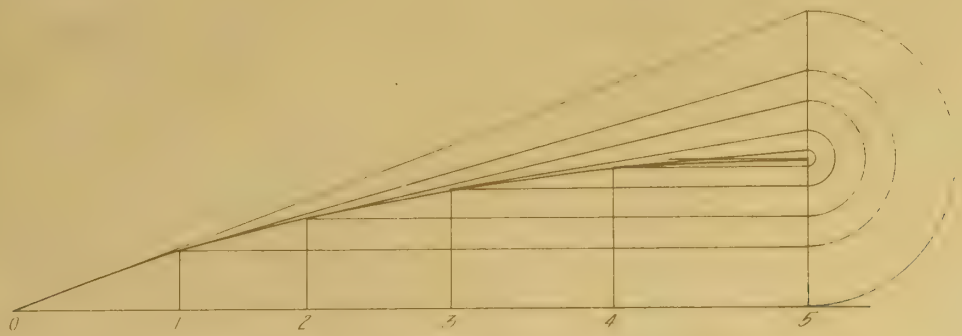
Theoretically, the truss acts as a two-hinged arch, the lower horizontal chord preventing deformation in the direction of the truss. The top chord corresponds closely to a true parabola, the greatest variation being at panel point 1, where the ordinate is six inches greater than the computed ordinate. The loads on the bridge are transferred



10 Panels – 100 Ft end to end Span

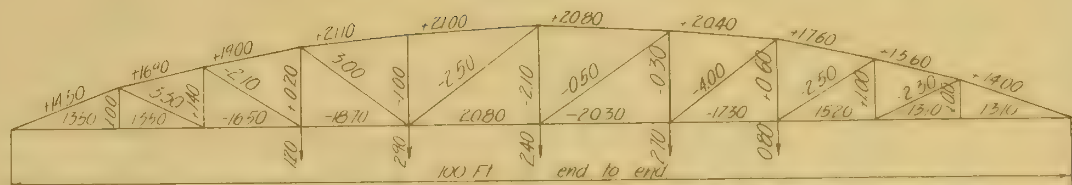
Joints assumed as hinged

MAXIMUM AND MINIMUM STRESSES.

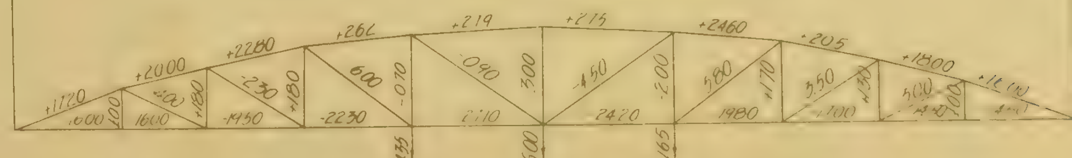


Tangents to Arch Rib at Panel Points

Thrusts at Panel Points in Arch Rib due to Dead Load



Stresses due to 2 Teams and 2 Heavily Loaded Wagons.



Stresses due to 24000-Lb. Traction Engine.

PLATE II

to the top chord by means of vertical hangers with nuts bearing on the top channel;

The Reactions for an Arch Rib.

In order to determine the thrust and stresses in any arch rib, the reactions at the supports must first be determined. A theoretical discussion and derivation of the formula for determining the reactions of a two-hinged arch rib will now be given.

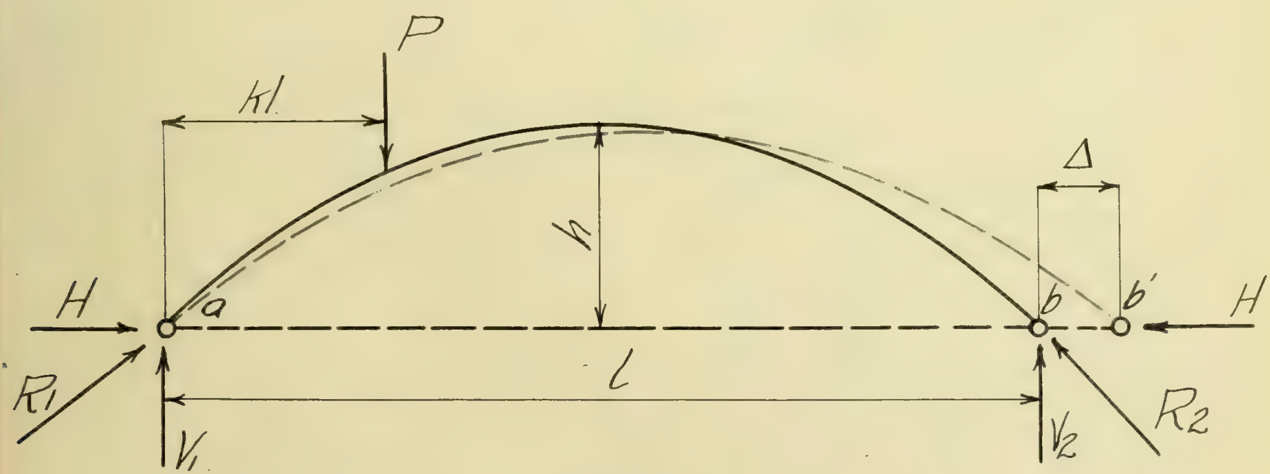


Fig. 1.

In Fig. 1, let l be the span of a two-hinged arch, and h the rise of its crown. Let a load P be placed any distance Kl from the left support, K being any fraction

less than unity. This load P then is held in equilibrium by the two inclined reactions R_1 and R_2, whose lines of action must intersect P at a common point. The reaction R_1 may be replaced by its horizontal and vertical components H and V_1, and likewise R_2 may be replaced by H and V_2. Here H is the horizontal thrust at the hinges due to the load P, and it is evidently the same at both hinges, because the sum of the horizontal forces acting on the structure must be equal to zero.

The vertical components of the reactions are the same as those for a simple beam, and are found by taking moments about the supports a and b. Then

$V_1 = P(1-K)$ $\qquad$ $V_2 = PK.$

The value of the horizontal component H can not be found by pure statics alone, for there are three unknown forces to be found while the principles of statics furnish but two conditions of equilibrium.

Let the arch rib in Fig. 1 be supposed to be placed on rollers at the end b so that when P causes a deflection of the rib the end b moves horizontally to b'. In this condition there is no thrust H. Let Δ represent the horizontal displacement bb'. Now suppose a horizontal force H to be applied at b' which is sufficiently large to bring b' back to b. Then the value of Δ due to P is equal to the value of Δ produced by H. This is the condition by which the horizontal thrust H is determined. By finding expressions (one of which contains the term H), of these two values of the displacement Δ, and equating these expressions Δ may be eliminated and H determined.

The deformation of an arch rib is due mainly to flexure. The flexural stresses, when the elastic limit is not exceeded, are proportional to their distances from a neutral surface, upon which there is no stress due to flexure. To find the horizontal displacement Δ due to P, let a hori-

zontal force unity be applied at b in the direction of bb'. The external work overcome in the displacement is

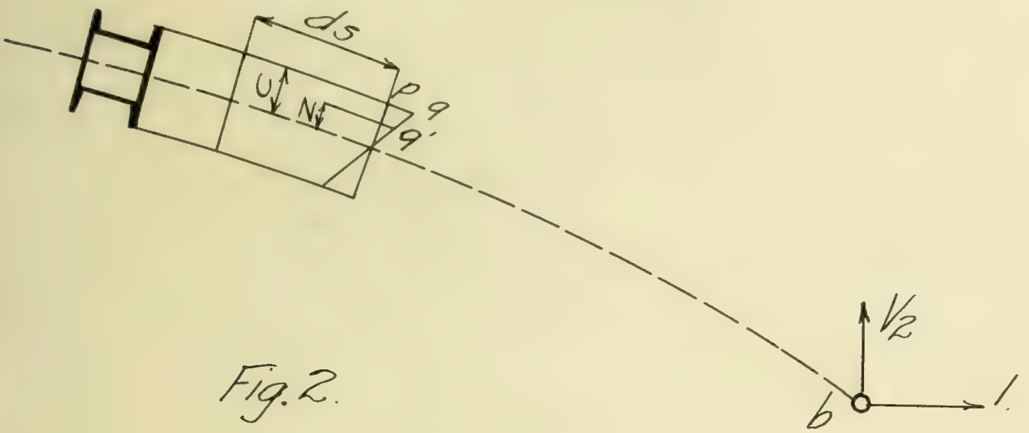


Fig. 2.

then, from the principles of mechanics, is equal to $\frac{1}{2}(P \times \Delta)$ or $\frac{1}{2}\Delta$, which is equal to the internal work of the flexural stresses. Let ds, Fig. 2, be an elementary length of the arch rib and I the moment of inertia of its cross section about the neutral axis. Let M' be the bending moment of the vertical forces, which produces a unit stress $\frac{M'c}{I}$ on the remotest fiber, c being the distance of that fiber from the neutral axis. From the principles of mechanics, the elongation or shortening of this fiber is given by $\lambda = \frac{SL}{E}$ or $\lambda = \frac{M'c}{I} \cdot \frac{ds}{E}$, which is represented by pq; and

that of any other fiber distant z from the neutral axis is $\frac{M'z}{I}, \frac{ds}{E}$. Now let m be the moment due to the horizontal force unity at b. From the fundamental formula $S = \frac{Mc}{I}$, the unit stress due to this on the fiber $p'q'$ is $\frac{mz}{I}$ and if a be the area of that fiber, the total stress on it is $\frac{maz}{I}$. Then from the general formula for internal work, $k = \frac{1}{2}P\lambda$, the internal work of this fiber is accordingly $K' = \frac{1}{2} \cdot \frac{maz}{I} \cdot \frac{M'z}{I} \cdot \frac{ds}{E} = \frac{M'maz^2ds}{2EI^2}$; and the summation of this over the entire cross section is effected by putting $\Sigma az^2 = I$. Then the total internal work done by the load P in the entire arch rib is given by

$$K = \int_0^l \frac{M'mds}{2EI}.$$

Equating this to the internal work gives

$$\frac{1}{2}\Delta = \int_0^l \frac{M'mds}{2EI}$$

Then

$$\Delta = \int_0^l \frac{M'mds}{EI} \qquad (1).$$

which is the horizontal displacement of b due to the effect of the vertical load P.

Now it remains to determine

the value of Δ due to the horizontal thrust H. Let M'' be the moment due to the horizontal thrust H. Then $M'' = -Hm$ since m was taken equal to the moment due to a unit horizontal force acting away from b. By similar reasoning as before,

$$\frac{1}{2} H\Delta \text{ (external work)} = \int_0^l \frac{M''^2 ds}{2EI} \text{ (internal work)}$$

Substituting $M'' = -Hm$ and solving for Δ we have

$$\Delta = H \int_0^l \frac{m^2 ds}{EI} \qquad (2)$$

Equating the two values of Δ expressed by equations (1) and (2) gives a condition such that the hinge b cannot move horizontally under the action of the load P.

$$\int_0^l \frac{M'm\,ds}{EI} = H \int_0^l \frac{m^2 ds}{EI}$$

$$\therefore \quad H = \frac{\int_0^l \frac{M'm\,ds}{EI}}{\int_0^l \frac{m^2 ds}{EI}} \qquad (3)$$

which is the general formula for determining the thrust for a two-hinged arch rib under the action of flexural stresses.

Thrust for a Parabolic Arch Rib.

Since the arch rib under investigation is approximately parabolic in form, the formula for the thrust of a parabolic two-hinged arch due to flexural stresses produced by a single load P will now be determined.

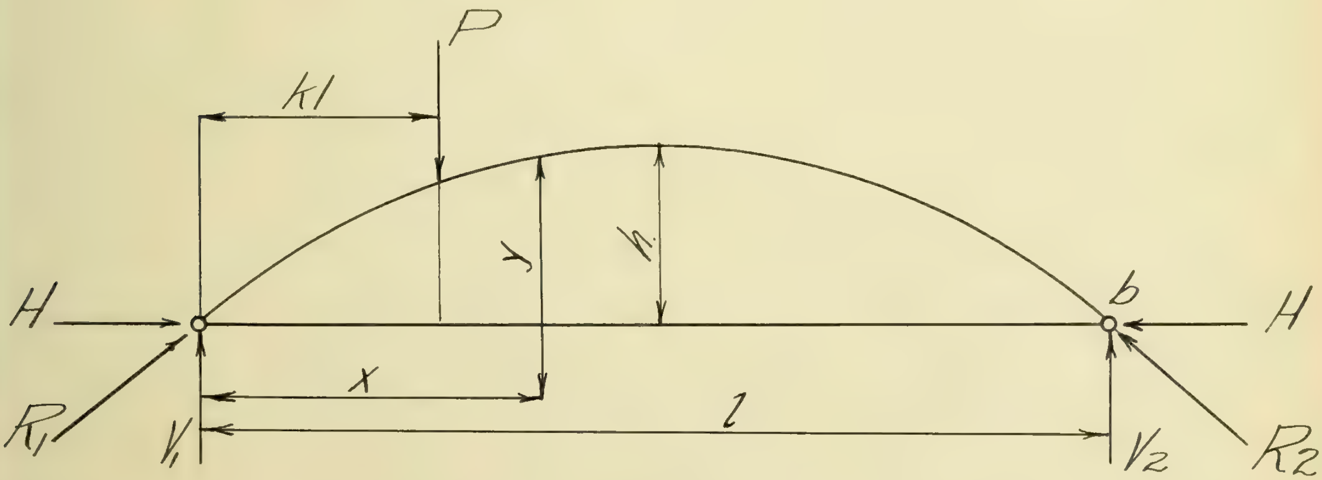


Fig. 3.

The equation of the parabola referred to the hinge a as origin is $y = 4h\left(\frac{x}{l} - \frac{x^2}{l^2}\right)$, from which the ordinates y may be computed for all values of x.

A single load P is placed on this arch at a distance kl from the left end. From the preceding discussion, the vertical reactions due to P are $V_1 = P(1-k)$

and $V_2 = Pk$. It is required to find the analytical value of the horizontal thrust H.

As has been stated before, the deformation under the action of exterior forces is due mainly to flexural stresses. At any section to the left of P the bending moment is $M = V_1x - Hy$, and at any section to the right of P it is $M = V_1x - P(x-kl) - Hy$. Let M' represent the bending moment due to the vertical forces, and M'' that due to the horizontal thrust H. Then M' has the value V_1x at any point from the left end to P, and the value $V_1x - P(x-kl)$ at any point from P to the right end. The value of M'' is $-Hy$. In general y is the moment due to a horizontal force of unity acting away from b and is equivalent to m in the preceding discussion. The total bending moment M due to all the external forces is given by $M = M' + M'' = M' - Hy$.

The general formula (3) can now be applied in determining the thrust H to a parabolic arch rib. In

the arch rib under consideration. the moment of inertia I of the rib cross section varies approximately from the crown to the hinges, as the secant of the angle of inclination of the axis of the rib. If I_c be the moment of inertia at the crown, then the moment of inertia at any point along the rib is $I = I_c \sec i$. From the fundamental theorem in calculus, $ds = dx \sec i$. The general formula (3) deduced in the preceding discussion is

$$H = \frac{\int_0^l \frac{M' m\, ds}{EI}}{\int_0^l \frac{m^2 ds}{EI}} \qquad (3)$$

The values to be substituted in the above equation are as follows:—

$M' = V_1 x$ or $V_1 x - P(x - kl)$

$V_1 = P(1-k)$

$ds = dx \sec i$

$I = I_c \sec i$

$m = y = 4h\left(\frac{x}{l} - \frac{x^2}{l^2}\right)$.

Then:—

$$H = \frac{\int_0^{kl} \frac{V_1 x \cdot y \cdot dx \sec i}{EI \sec i} + \int_{kl}^{l} \frac{[V_1 x - P(x - kl)]\, y \cdot dx \sec i}{EI \sec I}}{\int_0^l \frac{y^2 \cdot dx \cdot \sec i}{EI \sec i}}$$

Substituting the value for y, we get

$$H=\frac{\int_0^{kl} P(1-k)x\cdot 4h\left(\frac{x}{l}-\frac{x^2}{l^2}\right)\cdot dx+\int_{kl}^{l}\left[P(1-k)x-P(x-kl)\right]4h\cdot\left(\frac{x}{l}-\frac{x^2}{l^2}\right)dx}{\int_0^l 16h^2\left(\frac{x}{l}-\frac{x^2}{l^2}\right)^2dx}$$

and after simplifying the expression becomes

$$H=\frac{\frac{P}{4h}\int_0^{kl}\left(\frac{x^2}{l}-\frac{kx^2}{l}-\frac{x^3}{l^2}+\frac{kx^3}{l^2}\right)dx+\frac{P}{4h}\int_{kl}^{l}\left(kx-\frac{2kx^2}{l}+\frac{kx^3}{l^2}\right)dx}{\frac{P}{4h}\int_0^l\left(\frac{x^2}{l^2}-\frac{2x^3}{l^3}+\frac{x^4}{l^4}\right)dx}$$

integrating this we get

$$H=\frac{P}{4h}\cdot\frac{\left(\frac{x^3}{3l}-\frac{kx^3}{3l}-\frac{x^4}{4l^2}+\frac{kx^4}{4l^2}\right)_0^{kl}+\left(\frac{kx^2}{2}-\frac{2kx^3}{3l}+\frac{kx^4}{4l^2}\right)_{kl}^{l}}{\left(\frac{x^3}{3l^2}-\frac{x^4}{2l^3}+\frac{x^5}{5l^4}\right)_0^l}\quad\text{and}$$

between limits o and l its value is

$$H=\frac{P}{4h}\cdot\frac{\left(\frac{k^3l^2}{3}-\frac{k^4l^2}{3}-\frac{k^4l^2}{4}+\frac{k^5l^2}{4}+\frac{kl^2}{2}-\frac{2kl^2}{3}+\frac{kl^2}{4}-\frac{k^3l^2}{2}+\frac{2k^4l^2}{3}-\frac{k^5l^2}{4}\right)}{\left(\frac{l}{3}-\frac{l}{2}+\frac{l}{5}\right)}$$

which after combining and simplifying, becomes equal to

$$H=\frac{P}{4h}\cdot\frac{\left(\frac{k}{12}-\frac{k^3}{6}+\frac{k^4}{12}\right)l^2}{\frac{l}{30}}$$

or $H=\frac{5Pl}{8h}(k-2k^3+k^4).$ (4).

which is an expression for the horizontal

thrust of the parabolic two hinged arch rib due to flexural stresses produced by a single load P.

Therefore by utilizing this formula to compute the horizontal thrust at the hinges due to the action of each suspended load at the panel point, the total thrust becomes determinate by the summation of the separate thrusts.

The Computations of Reactions

The results of the computations for determining the reactions of the arch rib under consideration, or loads at various panel points are arranged in tabular form on the following page. The horizontal thrust at the hingls is computed from the formula $H = \frac{5Pl}{8h}(k-2k^3+k^4)$, the derivation of which has been given in the preceding pages. By substituting the values of l and h for this particular arch rib the above formula becomes

$$H = \frac{5 \times P \times 100}{8 \times 9.25}(k-2k^3+k^4) = 6.78\,P(k-2k^3+k^4).$$

TABLE III.
REACTIONS OF THE ARCH RIB.

No of Joint.	Joint Load Lb.	k.	V_1 Lb.	V_2 Lb.	$2k^3$.	k^4	H. lb.
1	2,370	0.105	2,120	250	0.0023	0.000	1,615
2	2,200	0.185	1,790	410	0.012	0.001	2,540
3	2,480	0.277	1,790	690	0.042	0.005	4,030
4	2,900	0.378	1,800	1,100	0.108	0.020	5,700
5	3,120	0.500	1,560	1,560	0.250	0.062	6,600
6	2,900	0.622	1,100	1,800	0.480	0.150	5,700
7	2,480	0.723	690	1,790	0.756	0.272	4,030
8	2,200	0.815	410	1,790	1.082	0.440	2,540
9.	2,370	0.895	250	2,120	1.432	0.640	1,615
Totals.			11,510	11,510		H =	34,370

The vertical reactions at the hinges due to dead load are 11,510 lb. at either end and the horizontal reactions are 34,370 lb. The resultant reaction R, is 36,300 lb. It will be noted by way of comparison that the value of 34,370 lb. for the horizontal thrust computed by this method compares closely with the stress in the horizontal tie as determined by graphical methods, Plate I. Also H, as computed for a parabolic arch rib under 256 lb. per lin. ft. dead load is $\frac{wl^2}{8h} = \frac{256 \times 100^2}{8 \times 9.25} = 34600$ lb. which serves as another check on the result and further shows that the arch rib differs little from a parabola.

Thrust at Panel Points.

With the preceding data, the thrust at the panel points can now be determined. The solution is best made by graphical methods as shown on Plate II. The thrust at any point acts in the direction of the tangent to the curve at that point. The tan-

gents are drawn by use of the condition that the subtangent to a parabola is bisected at the vertex. By projection on the tangent from the rays of the equilibrium polygon, the thrusts at the panel points are determined.

The Influence of Temperature.

The variations in temperature change the value of the horizontal thrust H, but do not affect V_1 or V_2. It is usually specified that an arch shall be designed to be subject to a variation of $\pm$ 75 degrees Fahrenheit.

Let ρ be the coefficient of expansion and t the rise in temperature then the span l will be increased by $\rho t l$, provided one end is free to move. As both hinges are fixed in position when the supports do not yield, equal and opposite positive reactions H_t are produced. The value of H must be such as to prevent the horizontal displacement $\rho t l$ which corresponds to Δ

in the preceding discussion. This value of Δ is $H\int \frac{my\,ds}{EI_c}$. The value of H is found by making $\Delta = \rho t l$, $m = y$, $ds = dx \sec i$ and $I = I_c \sec i$. Then

$$EI_c\rho t l = H\int_0^l y^2\,dx = \frac{16Hh^2}{l^4}\int_0^l (lx - x^2)^2\,dx$$

Integrating and solving for H, there results for a rise in temperature of $t°$ Fahrenheit

$$H_t = + \frac{15EI_c\rho t}{8h^2} \qquad (5)$$

Similarly for a fall in temperature,

$$H_t = - \frac{15EI_c\rho t}{8h^2}. \qquad (6)$$

For the arch rib under consideration $E = 25{,}000{,}000$ lb. per sq. in., $I_c = 69.2$ in.^{4}, $\rho = 0.0000067$, $t = \pm 75$ degrees Fahr. and $h = 111$ inches. Substituting these values the temperature thrust is found to be

$$H = \pm 1{,}470 \text{ lb.}$$

Since this value is less than 5% of the horizontal thrust due to dead load it will not be further considered in the investigation.

Stresses due to Wind.

The stresses in the lower lateral lateral system are computed for a un-

iform dead wind load of 150 lb., and for a live wind load of 150 lb., both per lin. ft. of truss. The oak floor joists are dapped 1 in. to 2 in. on the lower chord and act as the struts in the lateral system. The dead and live load wind stresses are given on a diagram Plate I.

Stresses due to Two Teams and Two Loaded Wagons

The stresses due to two teams of 3,200 lb. each and two heavily loaded wagons of 7,000 lb. each were determined by graphical methods. The loads were placed tandem over the center of the span as shown in Fig 5.

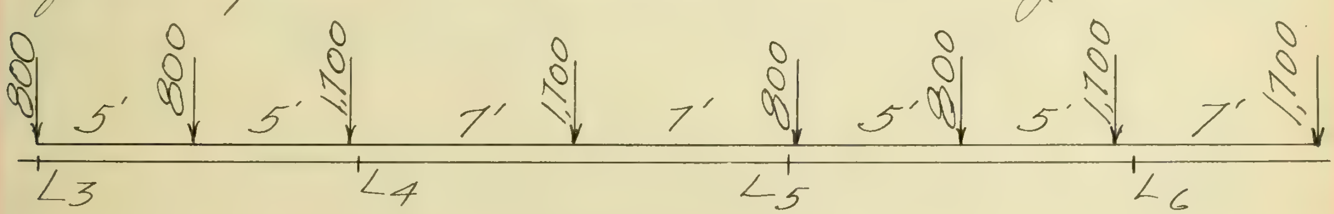


Fig.5.

The resulting stresses for the loads in this position are placed on the stress sheet of the truss, Pl. II.

Stresses due to a 24,000 lb. Traction Engine.

The stresses were determined graphically for a 24,000 lb. traction engine placed near the center of the span. The weight of the engine with 2 in. of water in the gauge-glass is 24,300 lb. The weight on the rear drivers is 16,340 lb, and on the front wheels is 6,180 lb.

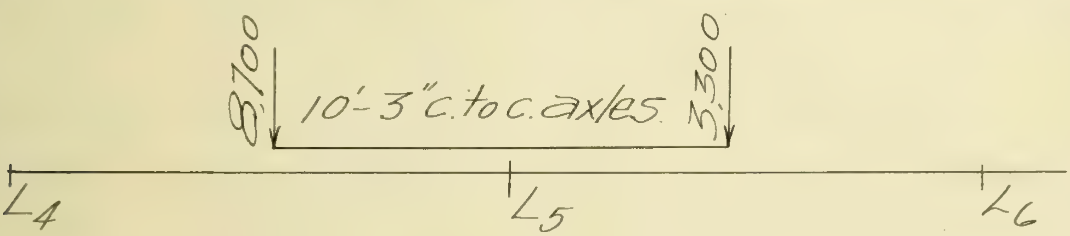


Fig. 6.

The load diagram for one truss and its position on the bridge is shown in Fig. 7. The resulting stresses are placed on a stress sheet of the truss, Pl. II.

Secondary Stresses in Lower Chord.

The peculiar manner of connecting the floor system to the lower chord by notching the joists over the two horizontal bars which comprise the lower chord, develop secondary stresses in

this member. The case is that of a simple beam carrying vertical loads in addition to a direct longitudinal tensile stress. The secondary stresses developed in the lower fibers of the chord are tensile, and those developed in the upper fibers are compressive. The secondary tensile stresses will be computed in a panel near the center, which is apparently the most dangerous section of the lower chord.

The weight of the floor per lineal ft. on one truss is 170 lb. The live load per lineal ft. of truss is $\frac{80 \times 14.5}{2} = 580$ lb. The total load per lineal ft. of truss is $170 + 580 = 750$ lb. This actually consists of a series of concentrated loads spaced about 2 ft. apart; but approximately the same result will be obtained by considering it as a uniform load. The forces acting on this beam are

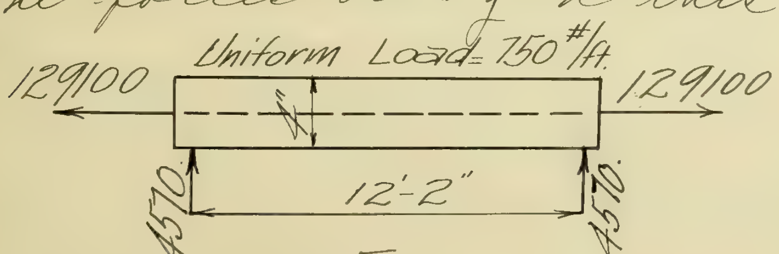


Fig. 7.

shown in Fig. 7. The direct stresses are;

D.L. = 34,100 lb; L.L. = 77000 lb, Wind = 18,000 lb, total = 129,100 lb. The reaction R is $\frac{12.2 \times 750}{2}$ = 4,570 lb. The maximum secondary fiber stress on the lower side of the bar is given by

$$S = \frac{My}{I + \frac{Pl^2}{10E}} \qquad (7.$$

The maximum moment M in pound-inches is $4{,}570 \times \frac{146}{2} - 4{,}570 \times \frac{146}{4} = 166{,}000$ lb-in; the distance from the neutral axis to the remotest fiber, $y = 2$ in; the moment of inertia about the neutral axis is $\frac{1}{12} bd^3 = \frac{1}{12} \times 1 \times 4^3 = 5.33$; the direct tensile stress P is 129,000; the length l is 146 in; E, the moment of inertia for wrought iron is 25,000,000 lb. per sq. in. Then

$$S = \frac{166000 \times 2}{5.33 + \frac{129000 \times 146^2}{10 \times 25000000}} = 20300 \text{ lb. per sq. in.}$$

which is the maximum secondary tensile stress in the lower chord due to the weight of the floor system and a live load of 80 lb. per sq. ft. of floor surface.

Similarly the secondary stress for the dead load alone is found to be 10,500 lb. per sq. in; for the dead load, two teams and two loaded wagons it is 19,000 lb. per sq. in;

and for the dead load and 2,000 lb. engine it is computed to be 71,000 lb. per sq. in.

The Efficiency of the Members.

The efficiency of the different members will be investigated by considering an allowable unit stress of 25,000 pounds per square inch for dead load and 12,500 pounds per square inch for live load. These high unit stresses are taken instead of the usual values for wrought iron of 20,000 and 10,000 respectively, because it is desired to give the bridge all the benefit possible in this investigation. Then, in case the results do not satisfy the requirements, it is certain that the bridge lacks the proper degree of safety.

The Lower Chord.

The lower chord is composed of two bars, 4" x $\frac{1}{2}$". The gross section at the center is reduced by a $\frac{5}{8}$-in. rivet for

the lateral connection. Since the maximum tensile stress occurs at the center of the span, it is here that this member is most likely to fail.

The maximum direct stresses are taken from Plates I and II. They are; D.L. = 34,100 lb., L.L. (for 80 lb. per sq. ft of floor surface) = 77,000 lb., and Wind = 18,000 lb. The allowable unit stresses are; D.L. 25,000 lb. per sq. in., L.L. 12,500 lb. per sq. in., and Wind = 18,000 lb. per sq. in. The area required for the D.L. stress is $\frac{34,100}{25,000} = 1.36$ sq. in. The area required for the L.L. stress is $\frac{77,000}{12,500} = 6.16$ sq. in. The area required for the wind stress is $\frac{18,000}{18,000} = 1$ sq. in. The total area required for the direct dead and live loads is 7.52 sq. in. The efficiency for the direct dead and live loads is the actual area ÷ the required area or $\frac{4.00 - \frac{3}{4} \times \frac{1}{2}}{7.52} = 48.2\%$. The efficiency for direct dead live and wind loads is $\frac{3.63}{8.52} = 42.6\%$.

The above efficiency does not consider the secondary stresses. They will now be considered. The secondary stress in the lower chord near its center has been computed and is 20,300 lb. per sq. in. for dead load,

uniform live load and wind load. The total L.L. unit stress is = 1200 + 30,300 = 41,500 lb. per sq. in. The direct D.L. unit stress is 1400 lb. per sq. in. The total wind unit stress is $\frac{1{,}800{,}000}{363}$ = 4,950 lb. per sq. in. Reducing to a basis of L.L. unit stress, $L_E = L + \frac{12{,}500}{25{,}000}D + \frac{12{,}500}{18{,}000}W = 41{,}500 + \frac{12{,}500}{25000} \times 9401 + \frac{12{,}500}{18000} \times 4{,}950 = 49{,}800$ lb. per sq. in. Then the efficiency for dead load, uniform live load and wind is $\frac{\text{Allowable L.L. unit stress}}{L_E} = \frac{12{,}500}{49{,}800} = 25.1\%$. Then in a similar manner the efficiency for dead load, traction engine and wind load is computed to be 14.8%. These values show the exceedingly unsafe condition of the lower chord.

The Efficiency of the Verticals.

Since all the verticals have the same cross section being circular rods $1\frac{1}{2}$ inches in diameter, the weakest vertical is that one having the largest stress. By referring to Plate II, this is found to be the middle vertical. The D.L. stress is 3,120 lb. and the L.L. stress, 7,010 lb. The area required for the D.L. stress is $\frac{3{,}120}{25{,}000} = 0.12$ sq. in. The

area required for the L.L. stress is $\frac{7,011}{12,500}$ = 0.56 sq. in. The total area required is 0.68 sq. in. The efficiency is the actual area ÷ by the required area or $\frac{2.25}{0.68}$ = 330% Therefore the verticals have ample section to withstand these stresses.

The Efficiency of the Diagonals.

All the diagonals are composed of one circular rod $\frac{3}{4}$ inches in diameter. The maximum stresses occur in the diagonal near the center. The D.L. stress is 1,120 lb. The L.L. stress is 12,500 lb. The area required for the D.L. stress is $\frac{1,120}{25000}$ = 0.04 sq. in. The area required for the L.L. stress is $\frac{12,500}{12500}$ = 1.00 sq. in. The total required area is 1.04 sq. in. The efficiency is $\frac{0.56}{1.04}$ = 54%, which shows that the diagonals have not sufficient section to meet the requirements.

The Efficiency of the Top Chord.

The most unsafe portion of the top chord is near the center of the span

as may be seen by reference to Plate I. This condition is due to the fact that while there is only a small variation in stress between the abutments and center of span, there is a comparatively large reduction in the cross section of the member.

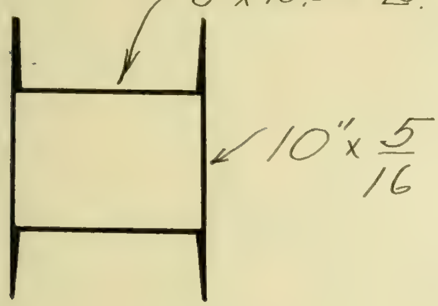


Fig. 8.

The area of section at center (Fig. 8) is 12.43 sq. in. The D.L. stress is 34,300 lb. and the L.L. stress is 77,500 lb. The length of member between panel points is 12 feet 6 inches. The allowable D.L. stress is $24000 - \frac{110\,l}{r} = 24{,}000 - \frac{110 \times 78}{2.96} = 21{,}100$ lb. per sq. in. The allowable L.L. stress is $\frac{1}{2} \times$ D.L. $= 10.550$ lb. per sq. in. The area required for the D.L. stress is $\frac{34300}{21{,}100} = 1.62$ sq. in. The area required for the L.L. stress is $\frac{77{,}500}{10{,}550} = 7.35$ sq. in. The total area required is 8.97 sq. in. The efficiency is the actual area ÷ by the required area or $\frac{12.43}{8.97} = 139\%$. Therefore the top chord has ample section to withstand these stresses.

The Efficiency of the Lower Lateral System.

All the diagonals in the lower lateral system consist of iron rods, $\frac{3}{4}$ in. in diameter. The maximum D.L. stress as theoretically determined is 7,960 lb. The L.L. stress is 7,960 lb. The total stress is 15,920 lb. The allowable unit stress for wind is 18000 lb. per sq. in. The area required is $\frac{15920}{18000} = 0.88$ sq. in. The efficiency is $\frac{\text{the actual area}}{\text{the required area}} = \frac{0.56}{0.88} = 64\%$. This is the theoretical efficiency, but it is almost certain that the lateral system as constructed does not form a system at all. Its place is taken by the floor system, which acts as a flat girder extending the entire length of the span, and thus transfers the wind stresses to the foundations.

Summary.

On the following pages are given tables for the purpose of comparison of the efficiency of the various members. The live load stresses considered in Table IV are those due to uniform live load of 80 lb. per sq. ft.

In order to more clearly show the deficiency of the carrying power of this bridge as compared with those which carry the usual live load of 80 lb. per sq. ft. of floor surface, the unit load, in the case of each member will be computed, which will stress that member up to the limit of its allowable unit stress. The computations follow and the results are tabulated in Table V.

In the case of the lower chord, the direct D.L. unit stress is $\frac{34,100}{3.62}$ = 9,400 lb. per sq. in. The secondary unit stress is 10,500 lb. per sq. in. The total D.L. unit stress is 9,400 + 10,500 = 19,900 lb. per sq. in. The allowable D.L. unit stress is 25,000 lb. per sq. in. Therefore, we have an excess allowable D.L. unit stress of 5,100 lb. per sq. in. in this member. Then the allowable L.L. unit stress is $\frac{5,100}{2}$ = 2,550 lb. per sq. in. Now it remains to determine what unit live load will cause this unit stress in the member. By employing the formula, $S = \frac{My}{I + \frac{Pl^2}{10E}}$, for the

computation of the secondary stress and by computing the direct stress by the theory of proportion, it is found after a number of trials, that the live load which will cause this unit stress is 20 lb. per sq. ft. of floor surface. The details of these computations are similar to what has been given before, p. 25, and will not be reproduced here.

In the case of the upper chord, the area required for the D. L. stress is $\frac{34,300}{21,100} = 1,62$ sq. in. The total area of section is 12,43 sq. in. Then the area available for live load stresses is $12,43 - 1,62 = 10,81$ sq. in. The allowable L.L. unit stress is 10,550 lb. per sq. in. The area required for the L.L. stress in this member caused by a load of 80 lb. per sq. ft. of floor surface is $\frac{77,500}{10,550} = 7,35$ sq. in. Then if x be taken as the allowable live load per sq. ft. of floor surface, we have by the theory of proportion, $\frac{x}{80} = \frac{10,81}{7,35}$, or $x = 117$ lb. per sq. ft. of floor surface.

Similarly, in the case of the diagonals, the area available for live load is 0.52 sq.in. The area required for the L. L. stress is 1.00 sq. in. The allowable live load is $\frac{80 \times 0.52}{1.00} = 41.6$ lb. per sq. ft. of floor surface.

Again, in the case of the verticals the area available for live load is 2.13 sq.in. The area required for the L. L. stress is 0.68 sq. in. The allowable live load is $80 \times \frac{2.13}{0.68} =$ 303 lb. per sq. ft. of floor surface.

TABLE IV.
SUMMARY OF RESULTS for DEAD LOAD AND UNIFORM LIVE LOAD

Member.	Actual Unit Stress. lb per sq.in.	Allowable Unit Stress. lb.per sq.in.	Excess. lb. per sq. in.	Factor of Safety.	Efficiency. %
Lower Chd.	52,310	15,000	-37,310	0.96	25.1
Upper Chd.	9,500	12,600	+3,100	5.26	139.0
Verticals.	4,500	15,000	+10,500	11.10	330.0
Diagonals.	24,200	15,000	- 9,200	2.03	54.0

TABLE V.
SUMMARY OF EFFICIENCIES FOR VARIOUS LOADINGS.

Member.	Uniform L.L. 80lb. per sq.ft. % Efficiency	Safe Uniform Live Load lb. per Sq.ft.	3,200-lb Team and 7,000-lb Wagon. % Efficiency.	24,000-lb Traction Engine. % Efficiency.
Lower Chd	25.1	2.0	38.8	14.8
Upper Chd	139.0	117.0	342.0	292.0
Verticals.	330.0	303.0	636.0	375.0
Diagonals.	54.0	41.6	156.0	117.0

∴ A safe traction engine would weigh 1,000lb.

The Conclusion.

The results of this investigation show that the bridge is in a very unsafe condition. The lower chord is an exceptionally weak member. The secondary stresses alone developed in this member are very much in excess of the safe working stresses. It is a fact that loads sufficiently heavy to have theoretically caused the failure of the structure have passed safely over the bridge. One explanation of this may be that the top chord acts as an arch rib, thus relieving the diagonals of excessive stress. Even in this case, the horizontal thrust at the hinges is evidently not all taken by the lower chord but by the masonry at the ends. Another explanation may be that the secondary stresses are of much less intensity than those which were computed. In those computations the lower chord was assumed to be a series of simple beams each of length

equal to that of a panel. The more exact condition is that of a continuous beam restrained at its supports. That the lower chord is continuous is a fact; but the restraining action of the castings at the panel points is evidently slight or otherwise the castings would have broken. These conditions would cause the unit stresses as determined for the lower chord and diagonals to be materially reduced, thus accounting for the fact that the ultimate strength of the structure has not been exceeded.